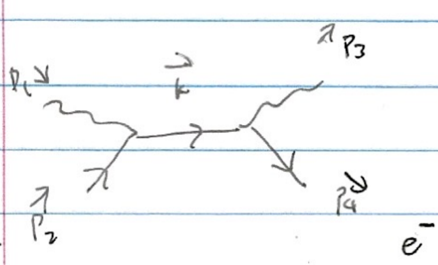
Schwartz
9.1 (a)

Consider $\bar{e}\gamma \to \bar{e}\gamma$: 



$$iM_1 = \varepsilon_1^\mu (-ie)(p_2+k)^\mu \frac{i}{k^2-m^2+i\varepsilon}(-ie)(k+p_4)^\nu \varepsilon_3^{*\nu} \times \delta^4(\cdots)$$

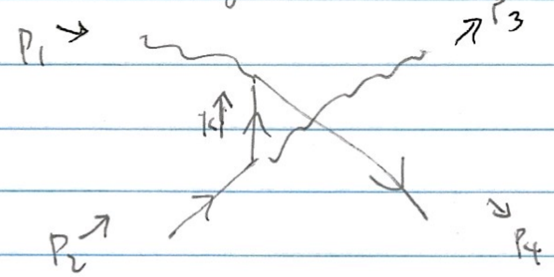
$$= \frac{i(ie)^2 \varepsilon_1^\mu (p_2+k)^\mu (k+p_4)^\nu \varepsilon_3^{*\nu}}{k^2-m^2+i\varepsilon} \times \delta^4(p_1+p_2-k)\,\delta^4(k-p_3-p_4)$$

$$= \frac{i(-ie)^2 \varepsilon_1^\mu (2p_2+p_1)(p_2+p_4+p_1)^\nu \varepsilon_3^{*\nu}}{(p_1+p_2)^2-m^2+i\varepsilon} \times \delta^4(\Sigma p)$$

Using $(p_1+p_2)^2-m^2 = p_1^2+p_2^2+2p_1p_2-m^2 = p_1^2+2p_1p_2$

$$M_1 = \frac{-e^2 \varepsilon_1^\mu (2p_2+p_1)^\mu (p_2+p_4+p_1)^\nu \varepsilon_3^{*\nu}}{p_1^2+2p_1p_2}$$

Now consider the diagram.

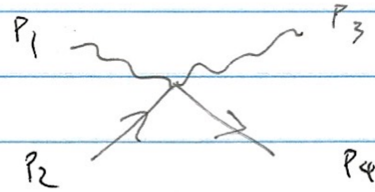


$$iM_2 = \varepsilon_1^\mu (-ie)(k+p_4)^\mu \frac{i}{k^2-m^2+i\varepsilon} (-ie)(p_2+k)^\nu \varepsilon_3^{*\nu} \times \delta^4(\cdots)\delta^4(\cdots)$$

Simplifying, and integrate over $\delta(p_1+k-p_4)$,

$$M_2 = \frac{-e^2 \varepsilon_1^\mu (2p_4-p_1)^\mu (p_2+p_4-p_1)\varepsilon_3^{*\nu}}{p_1^2-2p_1p_4} \times \delta^4(\Sigma p)$$

Lastly,



has contribution

$$iM_3 = 2ie^2 g_{\mu\nu} \varepsilon_1^\mu \varepsilon_3^{*\nu},$$

$$M_3 = 2e^2 g_{\mu\nu} \varepsilon_1^\mu \varepsilon_3^{*\nu}$$

$$\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 =$$

$$e^2 \varepsilon_1^{\mu} \left[2 g_{\mu\nu} - \frac{(2p_2 + p_1)^{\mu} (p_2 + p_4 + p_1)^{\nu}}{p_1^2 + 2p_1 p_2} - \frac{(2p_4 - p_1)^{\mu} (p_2 + p_4 - p_1)^{\nu}}{p_1^2 - 2p_1 p_4} \right] \varepsilon_3^{*\nu}$$

$$\times \delta^4(\Sigma p).$$

Replacing ε_1^{μ} with p_1^{μ}, we have

$$\mathcal{M}^{\mu} \propto \left[2p_1^{\nu} - \frac{(2p_2 p_1 + p_1^2)(p_2 + p_4 + p_1)^{\nu}}{p_1^2 + 2p_1 p_2} - \frac{(2p_4 p_1 - p_1^2)(p_2 + p_4 - p_1)^{\nu}}{p_1^2 - 2p_1 p_4} \right] \times \varepsilon_3^{*\nu}$$

$$= \left[2p_1^{\nu} - (p_2 + p_4 + p_1)^{\nu} + (p_2 + p_4 - p_1)^{\nu} \right] \times \varepsilon_3^{*\nu}$$

$$= \boxed{0.}$$

Davidson Cheng
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